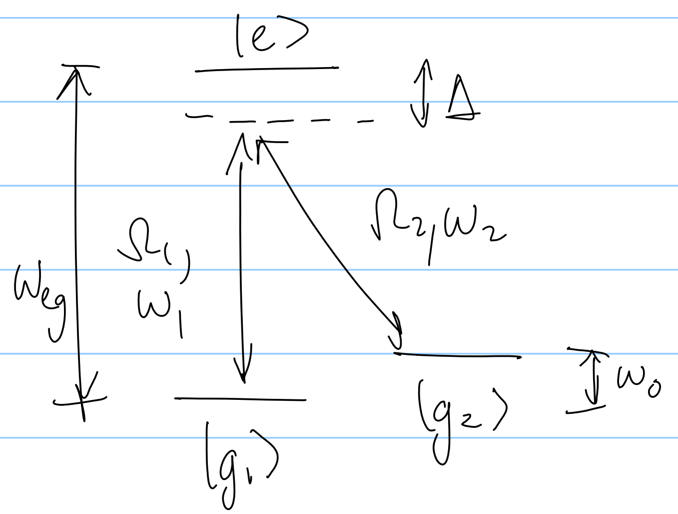
PHYS598MAP

Stimulated Raman transitions

Note Title 3/9/2005

We have to do a baby-version in class
(I have not seen the whole shebang outside of Wineland's personal notes)



<u>Assume</u>: ① field 1 only couples $|g_1\rangle$ and $|e\rangle$
② field 2 only couples $|g_2\rangle$ and $|e\rangle$
③ no other states

→ infinitely massive, $e^{i\vec{k}_i\cdot\vec{x}} \simeq 1$

→ we'll examine what happens in real life once we understand the general features of the physics first: note something simple:

$k_1, \hat\varepsilon = \hat x$

$k_2, \hat\varepsilon = \hat x$

$$\vec E_1 = E_1 e^{ik_1 z} e^{-i\omega_1 t}\,\hat x$$
$$\vec E_2 = E_2 e^{ik_2 z} e^{-i\omega_2 t}\,\hat x$$

$$\vec E = \vec E_1 + \vec E_2 = e^{ikz}\left(E_1 e^{-i\omega_1 t} + E_2 e^{-i\omega_2 t}\right)\hat x$$

$$|E|^2 = E_1^2 + E_2^2 + \underbrace{2E_1E_2\cos[(\omega_1-\omega_2)t]}$$

a beat at $\omega_1-\omega_2$, with strength $\propto E_1E_2$ — will this drive transitions between $|g_1\rangle$ and $|g_2\rangle$ — yes!

Now, do the quantum mechanics

$$\vec E_1 = \hat\varepsilon_1 E_1\left(e^{-i\omega_1 t} + e^{i\omega_1 t}\right)$$
$$\vec E_2 = \hat\varepsilon_2 E_2\left[e^{-i(\omega_2 t-\varphi)} + e^{i(\omega_2 t-\varphi)}\right]$$

interaction picture, interaction Hamiltonian:

$$H_I' = -e/2 \{ \langle e|\hat{\varepsilon}_1 \cdot \vec{r}|g_1\rangle |e\rangle\langle g_1| e^{i\omega_{eg}t} e^{-i\omega_1 t} E_1 + \langle e|\hat{\varepsilon}_2 \cdot \vec{r}|g_2\rangle |e\rangle\langle g_2| e^{i\omega_{eg}t} e^{-i(\omega_2 t - \varphi)} e^{-i\omega_0 t} E_2 \} + h.c.$$

$$\left.\begin{array}{l} \Delta = \omega_{eg} - \omega_1 \\ \Delta + \delta = \omega_{eg} - \omega_2 - \omega_0 \end{array}\right\}$$ there are two detunings now

Herr doktor Schrödinger's equation:

$$|\psi\rangle = c_1|g_1\rangle + c_2|g_2\rangle + c_e|e\rangle$$

$$i\hbar \frac{\partial\psi}{\partial t} = H_I'|\psi\rangle$$

$$\begin{cases} \dot{c}_1 = i\frac{eE_1^*}{2\hbar} e^{-i\Delta t} \langle g_1|\hat{\varepsilon}_1 \cdot \vec{r}|e\rangle c_e \\ \dot{c}_2 = i\frac{eE_2^*}{2\hbar} e^{-i\varphi} e^{-i(\Delta+\delta)t} \langle g_2|\hat{\varepsilon}_2 \cdot \vec{r}|e\rangle c_e \\ \dot{c}_e = i\frac{e}{2\hbar}\left[e^{i\Delta t}\langle e|\hat{\varepsilon}_1 \cdot \vec{r}|g_1\rangle E_1 c_1 + e^{i(\Delta+\delta)t} e^{i\varphi} \langle e|\hat{\varepsilon}_2 \cdot \vec{r}|g_2\rangle E_2 c_2\right] \end{cases}$$

define "couplings":

$$\boxed{\begin{aligned}\Omega_1 &= \frac{eE_1}{2\hbar}\langle e|\hat{\varepsilon}_1\cdot\vec{r}|g_1\rangle\\ \Omega_2 &= \frac{eE_2}{2\hbar}\langle e|\hat{\varepsilon}_2\cdot\vec{r}|g_2\rangle\end{aligned}}$$

and use a rotating frame:

$$c_e' = e^{-i\Delta t}c_e, \quad c_1' = c_1, \quad c_2' = c_2$$

so: $c_e = e^{i\Delta t}c_e'$

then: $\dot{c}_e = e^{i\Delta t}\dot{c}_e' + i\Delta e^{i\Delta t}c_e'$

assume $\dot{c}_e' \ll \Delta c_e'$

or $\Delta \gg \frac{\dot{c}_e'}{c_e'}$

huh? why?

- well, if $\Delta \gg \Omega_{1,2}$, then $\Omega' \simeq \Delta \longrightarrow$ oscillations in c_e are small and occur at frequency Δ
- and, in the rotating frame we remove all time dependence at Δ!

So, this condition is pretty good!

Then:

$$c_e' = \frac{1}{\Delta}\left[\Omega_1 c_1 + \Omega_2 e^{i\varphi} e^{i\delta t} c_2\right]$$

and

$$\begin{cases} \dot{c}_1 = \underbrace{\frac{i|\Omega_1|^2}{\Delta} c_1}_{\text{energy shift — because } |e\rangle \text{ really is there!}} + \underbrace{i\frac{\Omega_1^* \Omega_2}{\Delta} c_2 e^{i\delta t} e^{i\varphi}}_{\text{oscillations between } |g_1\rangle \text{ and } |g_2\rangle} \\ \dot{c}_2 = i\frac{|\Omega_2|^2}{\Delta} c_2 + i\frac{\Omega_2^* \Omega_1}{\Delta} c_1 e^{-i\delta t} \end{cases}$$

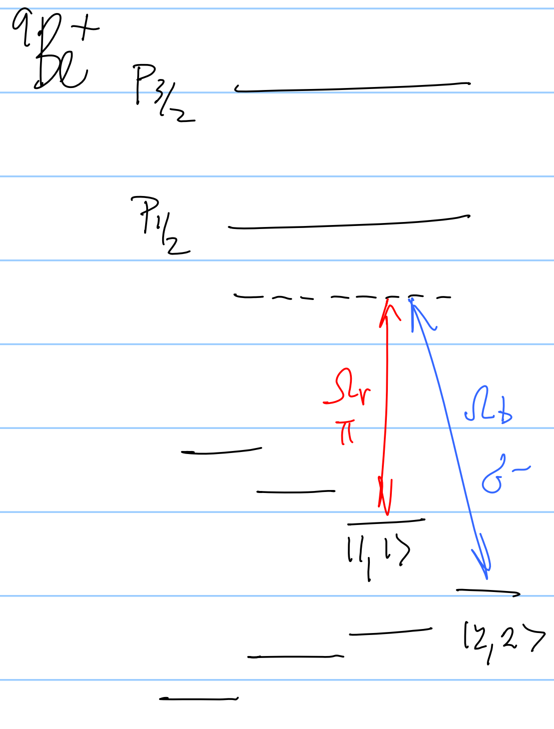
oscillations between $|g_1\rangle$ and $|g_2\rangle$ at Rabi rate $\Omega = \frac{\Omega_1 \Omega_2^*}{\Delta}$ with detuning δ

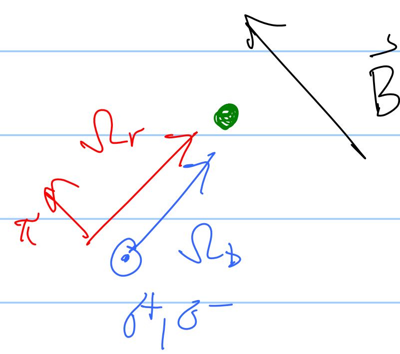
phase between laser fields controls the direction of the effective torque!

Voila! We get a coupled, 2-level system with $|g_1\rangle$ and $|g_2\rangle$

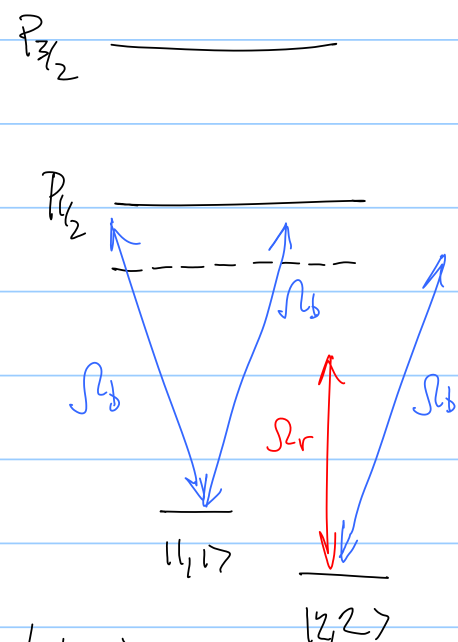
<u>What did we leave out?</u>

Many energy shifts Ω_1 will couple to $|g_2\rangle$ and vice-versa, but will not induce transitions





<u>additional energy shifts:</u>



but see... no additional coupling between $|1,1\rangle$ and $|2,2\rangle$

(remember to include <u>all</u> excited states)

$\boxed{\text{Also — should be clear now that } \Delta m_F = q_1 + q_2}$

(there are <u>real</u> excited states involved)

<u>What about the motion:</u>

<u>2 common cases:</u> I free states

II harmonic oscillator states

I $H_0 = \underbrace{\frac{\hbar^2 k^2}{2m}}_{\text{this is new!}} + \hbar\omega_0 |g_2\rangle\langle g_2| + \hbar\omega_{eg} |e\rangle\langle e|$

and we write $|\psi\rangle = \sum_k \left\{ c_{1k}|g_1\rangle + c_{2k}|g_2\rangle + c_{ek}|e\rangle \right\} |k\rangle$

<u>Our equations of motion look different now:</u>

$$\dot{c}_{1\vec{k}} = i\frac{eE_1^*}{2\hbar} e^{-i\Delta t} \sum_{\vec{k}'} e^{i(E_k - E_{k'})/\hbar\, t} \langle g_1 \vec{k} | \hat{\varepsilon}_1 \cdot \vec{r} e^{-i\vec{k}_1\cdot\vec{r}} | e\vec{k}' \rangle c_{e\vec{k}'}$$

$$\dot{c}_{2\vec{k}} = \frac{ieE_2^*}{2\hbar} e^{-i\varphi} e^{-i(\Delta+\delta)t} \sum_{\vec{k}'} e^{i(E_k - E_{k'})/\hbar\, t} \langle g_2 \vec{k} | \hat{\varepsilon}_2 \cdot \vec{r} e^{-i\vec{k}_2\cdot\vec{r}} | e\vec{k}' \rangle c_{e\vec{k}'}$$

... and a similar equation for $\dot{c}_{e\vec{k}}$

So, if $k_1, k_2 \ll 1/a_0$ but $k_1, k_2 \sim k$, then the motional state can change but the forces on the atomic scale are small!

$$\langle g_1 \vec{k} | \hat{\varepsilon}_1 \cdot \vec{r} e^{-i\vec{k}_1\cdot\vec{r}} | e\vec{k}' \rangle \longrightarrow \langle g_1 | \hat{\varepsilon}_1 \cdot \vec{r} | e \rangle \underbrace{\langle \vec{k} | e^{-i\vec{k}_1\cdot\vec{r}} | \vec{k}' \rangle}$$

$$\langle \vec{k} | \vec{k}' - \vec{k}_1 \rangle = \delta_{\vec{k}, \vec{k}' - \vec{k}_1}$$

and $\sum_{\vec{k}'} e^{i(E_k - E_{k'})/\hbar\, t} \langle g_1 \vec{k} | \hat{\varepsilon}_1 \cdot \vec{r} e^{-i\vec{k}_1\cdot\vec{r}} | e\vec{k}' \rangle c_{e\vec{k}'} \longrightarrow$

$$e^{-i \underbrace{\hbar^2 k_1^2 / 2m}_{\text{"recoil energy"}} t} \langle g_1 | \hat{\varepsilon}_1 \cdot \vec{r} | e \rangle c_{e, \vec{k}' - \vec{k}_1}$$

etc.

Being extremely careful, and doing the adiabatic elimination...!

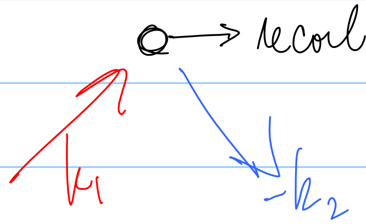
$$\dot{c}_{1,\vec{k}} = \frac{i|\Omega_1|^2}{\Delta} c_{1,\vec{k}} + i\frac{\Omega_1^* \Omega_2}{\Delta} e^{i\delta t} e^{i\varphi} e^{i\frac{\hbar}{2m}(k_2^2 - k_1^2)t} c_{2,\vec{k}+\vec{k}_1-\vec{k}_2}$$

$$\dot{c}_{2,\vec{k}+\vec{k}_1-\vec{k}_2} = i\frac{|\Omega_2|^2}{\Delta} c_{2,\vec{k}+\vec{k}_1-\vec{k}_2} + i\frac{\Omega_2^* \Omega_1}{\Delta} e^{-i\delta t} e^{-i\frac{\hbar}{2m}(k_2^2 - k_1^2)t} c_{1,\vec{k}}$$

We are coming close to being unable to escape the existence of photons!

$\vec{k}$ changes by $\vec{k}_1 - \vec{k}_2$ when going from $|1\rangle \to |2\rangle$

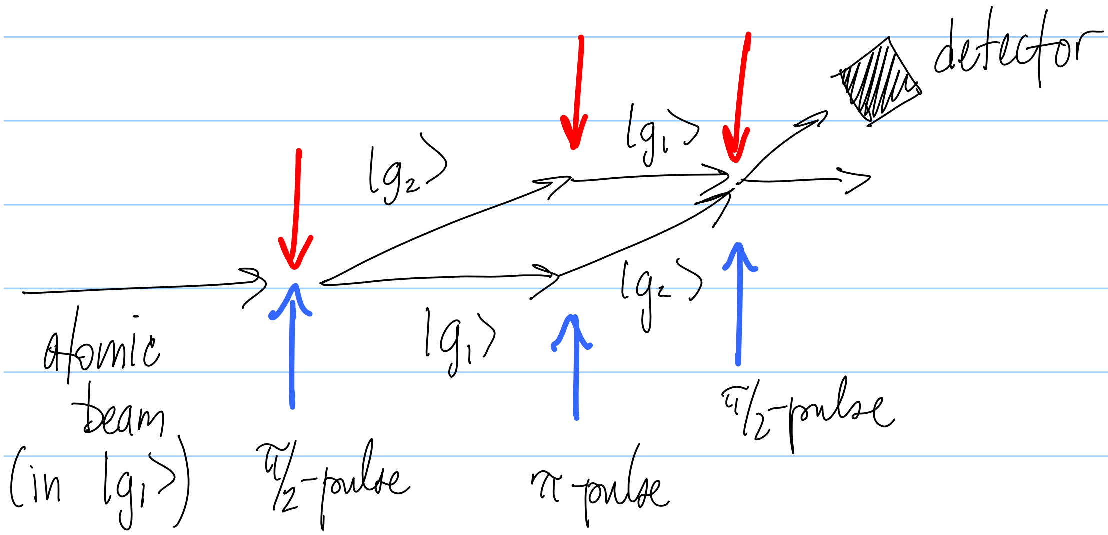
photon picture (although we didn't use them here):



absorb from $\vec{k}_1$, then emit into $\vec{k}_2$

Note that there is a spatially dependent field along the direction of recoil ("walking standing wave") — this is actually what exerts a force on the atom & changes motion

This trick is used in atom interferometers:



The most sensitive rotation-sensor in the world is made this way!

see Class. Quantum. Grav. 17, 2385 (2000)

In 1 sec, they get a sensitivity of better (these days) than $8\times10^{-6}\,\Omega_E$, where Ω_E is the Earth's rotation rate

(II) Life is more complicated when (1-D H_0):

$$H_0 = \hbar\omega |n\rangle\langle n| + \hbar\omega_0 |g_2\rangle\langle g_2| + \hbar\omega_{eg} |e\rangle\langle e|$$

$$|\psi\rangle = \sum_n \left(c_{1n}|g_1\rangle + c_{2n}|g_2\rangle + c_{en}|e\rangle \right)|n\rangle$$

Similar analysis, but $|n\rangle$ are not momentum eigenstates

the result:

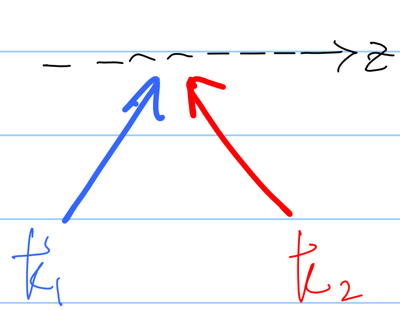
transitions $|g_1\rangle|n\rangle \longleftrightarrow |g_2\rangle|n'\rangle$ are possible

with Rabi rate: $\Omega = \frac{\Omega_1 \Omega_2^*}{\Delta} \langle n'| e^{i\Delta k\cdot x} |n\rangle$

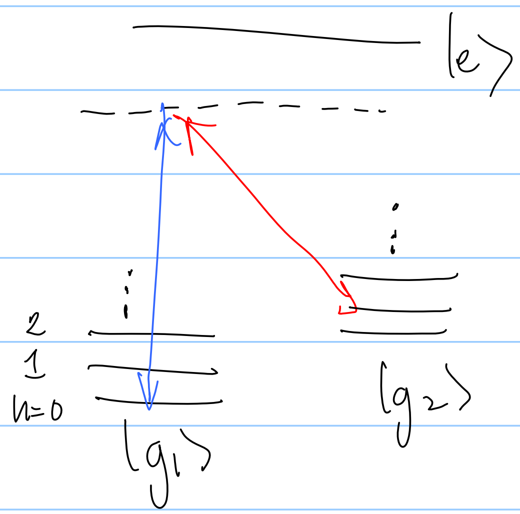
$$= \frac{\Omega_1 \Omega_2^*}{\Delta} \langle n'| e^{i\eta(a+a^\dagger)} |n\rangle$$

where $\eta = \Delta k_z \cdot z_0$ is the Lamb-Dicke parameter

($z_0 = \sqrt{\frac{\hbar}{2m\omega_z}}$ is the harmonic oscillator length)



$\Delta \vec k = \vec k_1 - \vec k_2$ wave vector difference



In the Lamb-Dicke limit, as $\Delta n = n'-n$ gets bigger, the Rabi rate is strongly suppressed

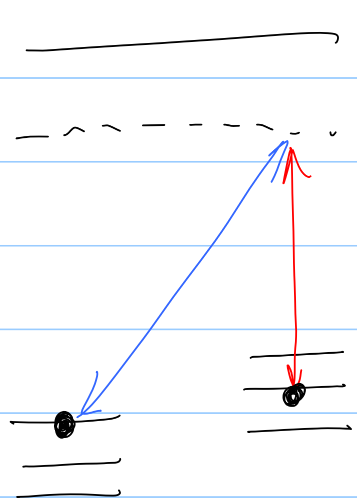
$$\langle n'| e^{i\Delta k_z z}|n\rangle = e^{-\eta^2/2}\sqrt{\frac{n_<!}{n_>!}}\,\eta^{|n'-n|}\,\underbrace{L_{n_<}^{|n'-n|}(\eta^2)}_{\text{generalized Laguerre polynomial}}$$

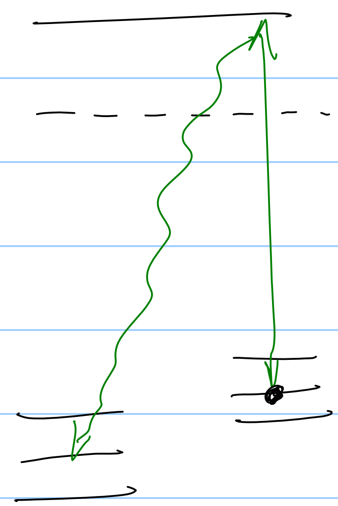
extremely similar to what is seen in x-ray Bragg scattering – "DeBye-Waller factor"

Stimulated Raman cooling:

We can cool an ion to $T=0$!

Multi-stage cooling process:





① π-pulse $|g_1 n\rangle \to |g_2, n-1\rangle$

② pump back using spontaneous emission — in the LD limit:

$|g_2, n-1\rangle \to |g_1, n-1\rangle$

③ repeat!